

### Summary of Asymptotic Notations

Asymptotic Bound	$\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = C$	Complete Definition	Asymptotic Family Relationships	Is $g(n)$ an asymptotically tight bound for $f(n)$ ?
$\omega$	$\infty = C$	$\omega(g(n)) = \left\{ \begin{array}{l} f(n) : \text{for any positive constant} \\ c > 0, \exists \text{ a constant } n_0 > 0 \text{ such that} \\ 0 \leq cg(n) < f(n) \forall n \geq n_0 \end{array} \right\}$	$\omega(g(n)) \subset \Omega(g(n))$	No; asymptotically larger (see pg 52)
$\Omega$	$0 < C \leq \infty$	$\Omega(g(n)) = \left\{ \begin{array}{l} f(n) : \exists \text{ positive constants } c \text{ and } n_0 \\ \text{such that } 0 \leq cg(n) \leq f(n) \forall n \geq n_0 \end{array} \right\}$	$\Theta(g(n)) \subseteq \Omega(g(n))$	Maybe
$o$	$0 = C$	$o(g(n)) = \left\{ \begin{array}{l} f(n) : \text{for any positive constant} \\ c > 0, \exists \text{ a constant } n_0 > 0 \text{ such} \\ \text{that } 0 \leq f(n) < cg(n) \forall n \geq n_0 \end{array} \right\}$	$o(g(n)) \subset O(g(n))$	No; asymptotically smaller (see pg 52)
$O$	$0 \leq C < \infty$	$O(g(n)) = \left\{ \begin{array}{l} f(n) : \exists \text{ positive constants } c \text{ and } n_0 \\ \text{such that } 0 \leq f(n) \leq cg(n) \forall n \geq n_0 \end{array} \right\}$	$\Theta(g(n)) \subseteq O(g(n))$	Maybe
$\Theta$	$0 < C < \infty$	$\Theta(g(n)) = \left\{ \begin{array}{l} f(n) : \exists \text{ positive constants } c_1, c_2, \text{ and } n_0 \\ \text{such that } 0 \leq c_1g(n) \leq f(n) \leq c_2g(n) \forall n \geq n_0 \end{array} \right\}$	Theorem 3.1 For any two functions $f(n)$ and $g(n)$ , we have $f(n) = \Theta(g(n))$ iff $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$	Yes

**Notes:**

$f(n)$  must be nonnegative whenever  $n$  is sufficiently large (asymptotically nonnegative)

$g(n)$  must be asymptotically nonnegative

if  $\forall n \geq n_0$  the function  $f(n)$  is equal to  $g(n)$  to within a constant factor, then  $g(n)$  is an asymptotically tight bound for  $f(n)$